IMPORTANT QUESTIONS / PROBLEMS FROM UNIT 1 OF SIGNALS AND SYSTEMS

1) All the theory questions and derivations given in the printed notes.
2) If $z(t) = x(t)y(t)$ find whether z(t) is even or odd signal?
   - x(t) = t sint y(t) = cost
   - x(t) = $2t\ sint\ cos^3 t$ y(t) = $3t^4\ sin^3 t\ cos^3 t$
   - x(t) = $sint\ cos^3 t$ y(t) = $t^6\ sin^2 t\ cos^3 t$
   - x(t) = $2t\ sint$ y(t) = $sin^3 t\ cos^3 t$
   - x(t) = $t\ sint\ cos^3 t$ y(t) = $t^{14}\ sin^{31} t\ cos^{39} t$
3) Find the period of the following signals
   - $x[n] = 25\ cos\left[\frac{\pi n}{3}\right] + 30\ sin^2\left[\frac{\pi n}{4}\right]$
   - $x[n] = 2\ cos\left[\frac{\pi n}{9}\right] + 3\ sin\ \left[\frac{n}{6}\right]$
   - $x[n] = sin\ \left[\frac{\pi n}{3}\right] + 6\ cos^2\left[\frac{\pi n}{16}\right]$
   - x(t) = $sin\ \left[\frac{\pi t}{36}\right] + sin\ \left[\frac{\pi t}{9}\right]$
   - x(t) = $sin\ \left[\frac{t}{36}\right] + sin\ \left[\frac{\pi t}{2}\right]$
4) Find and plot the even and odd components of the following signals.
   - $x[n] = \delta[n+3] + 2\delta[n+2] + 3\delta[n+1] + 4\delta[n] + 5\delta[n-4]$
   - $x[n] = 2\delta[n+6] + 6\delta[n+4] + \delta[n+2] + 4\delta[n] + 5\delta[n-6]$
   - $x[n] = 2\delta[n] + 6\delta[n-1] + \delta[n-2] + 2\delta[n-3] + 5\delta[n-4]$
   - x(t) = u(t+4) + u(t+2) + 2 u(t+1) + δ(t) – 2 u(t-1) – u(t-2) – u(t-4)
   - x(t) = u(t)
   - x(t) = sgn(t)
5) Plot the following signals, then find and plot their first derivatives.
   - $x(t) = 4u(t+3) - 8\,u(t-3) + 4\,u(t-5)$
   - $x(t) = -2u(t+4) + 8\,u(t-1) - 2\,u(t-9)$
6) Solve the following integrals?
   - $\int_{-\infty}^{\infty} t\ \delta(t+4)dt$
   - $\int_{-1}^{6} e^{-j2t}\ \delta(3t)\ u(t-1)dt$
   - $\int_{1}^{4}(t^2 + 9)\ \delta\ (t+3)dt$
   - $\int_{-\infty}^{\infty} e^{2t}\ \delta(2t)\ dt$
   - $\int_{-\infty}^{t} 2\delta(t-1)dt$
7) Find whether the following signals are energy signals or power signals?

(a) $x(t) = e^{-at}u(t),\quad a > 0$	(b) $x(t) = A\cos(\omega_0 t + \theta)$
(c) $x(t) = tu(t)$	(d) $x[n] = (-0.5)^n u[n]$
(e) $x[n] = u[n]$	(f) $x[n] = 2e^{j3n}$

8) Verify all the properties of the following systems?
   - y(t) =5x(t) + 5
   - y(t) = $x^2(t)$
   - y(t) = x(t) cost
   - y(t) = x(kt); k is a constant
   - y[n] = x[n-1]
9) For the given x[n] = $\delta(n-2) + 2\delta(n-1) + 3\delta(n) - 3\delta(n+1) - 2\delta(n+2) - \delta(n+3)$
   plot the following signals

- **X[n]** $\delta(n)$
- **X[n-2]** $\delta(n)$
- **X[3n+1]** $\delta(n-1)$
- **X[-n]** $\delta(n-2)$
- **X[n]** $u(n)$
- **X[n/2]** $u(n+2)$
- **X[2n-1] u(2n)**
- **X(n) { u(n)-u(-n)}**
- **-x(n) u(n)**
- **2X(n) u(-2n-2)**

10) For the given x(t) = $x(t) = 4u(t+3) - 8u(t-3) + 4u(t-5)$

plot the following signals

- **X(t) u(-t)**
- **X(2t+3) u(-t+2)**
- **X(t/3) u(-3+t)**
- **X(3t) u(t)**
- **X(-t) u(2t)**
- **X(t)** $\delta(t)$
- **X(t-1)** $\delta(t)$
- **X(t)** $\delta(t+1)$
- **X(t-2) u(t+2)**
- **X(t-2)** $\delta(t+2)$

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Solved problems:

Consider the capacitor shown in Fig. 1-33. Let input $x(t) = i(t)$ and output $y(t) = v_c(t)$.

a) Find the input-output relationship.

b) Determine whether the system is (*i*) memoryless, (*ii*) causal, (*iii*) linear, (*iv*) time-invariant, or (*v*) stable.

a) Assume the capacitance C is constant. The output voltage $y(t)$ across the capacitor and the input current $x(t)$ are related by [Eq. (*1.106*)]

$$y(t) = \mathbf{T}\{x(t)\} = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau \qquad (1.108)$$

b) (*i*) From Eq. (*1.108*) it is seen that the output $y(t)$ depends on the past and the present values of the input. Thus, the system is not memoryless.

(*ii*) Since the output $y(t)$ does not depend on the future values of the input, the system is causal.

(*iii*) Let $x(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$. Then

$$y(t) = \mathbf{T}\{x(t)\} = \frac{1}{C}\int_{-\infty}^{t} [\alpha_1 x_1(\tau) + \alpha_2 x_2(\tau)]\, d\tau$$

$$= \alpha_1\left[\frac{1}{C}\int_{-\infty}^{t} x_1(\tau)\, d\tau\right] + \alpha_2\left[\frac{1}{C}\int_{-\infty}^{t} x_2(\tau)\, d\tau\right]$$

$$= \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

Thus, the superposition property is satisfied and the system is linear.

(*iv*) Let $y_1(t)$ be the output produced by the shifted input current $x_1(t) = x(t - t_0)$. Then

$$y_1(t) = \mathbf{T}\{x(t - t_0)\} = \frac{1}{C}\int_{-\infty}^{t} x(\tau - t_0)\, d\tau$$

$$= \frac{1}{C}\int_{-\infty}^{t - t_0} x(\lambda)\, d\lambda = y(t - t_0)$$

Hence, the system is time-invariant.

Fig. 1-33

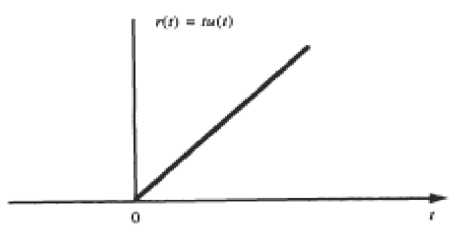

Fig. 1-34 Unit ramp function.

(*v*) Let $x(t) = k_1 u(t)$, with $k_1 \neq 0$. Then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} k_1 u(\tau)\, d\tau = \frac{k_1}{C}\int_{0}^{t} d\tau = \frac{k_1}{C} t u(t) = \frac{k_1}{C} r(t)$$

where $r(t) = tu(t)$ is known as the *unit ramp* function (Fig. 1-34). Since $y(t)$ grows linearly in time without bound, the system is not BIBO stable.

Consider the system shown in Fig. 1-35. Determine whether it is (*a*) memoryless, (*b*) causal, (*c*) linear, (*d*) time-invariant, or (*e*) stable.

(*a*) From Fig. 1-35 we have

$$y(t) = \mathbf{T}\{x(t)\} = x(t)\cos\omega_c t$$

Since the value of the output $y(t)$ depends on only the present values of the input $x(t)$, the system is memoryless.

(*b*) Since the output $y(t)$ does not depend on the future values of the input $x(t)$, the system is causal.

(*c*) Let $x(t) = \alpha_1 x(t) + \alpha_2 x(t)$. Then

$$\begin{aligned} y(t) = \mathbf{T}\{x(t)\} &= [\alpha_1 x_1(t) + \alpha_2 x_2(t)]\cos\omega_c t \\ &= \alpha_1 x_1(t)\cos\omega_c t + \alpha_2 x_2(t)\cos\omega_c t \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

Thus, the superposition property (*1.68*) is satisfied and the system is linear.

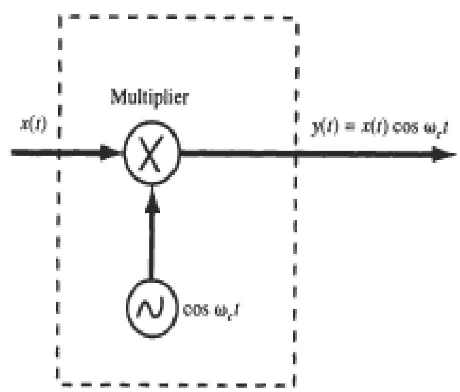

Fig. 1-35

(*d*) Let $y_1(t)$ be the output produced by the shifted input $x_1(t) = x(t - t_0)$. Then

$$y_1(t) = \mathbf{T}\{x(t - t_0)\} = x(t - t_0)\cos\omega_c t$$

But

$$y(t - t_0) = x(t - t_0)\cos\omega_c(t - t_0) \neq y_1(t)$$

Hence, the system is not time-invariant.

(*e*) Since $|\cos\omega_c t| \le 1$, we have

$$|y(t)| = |x(t)\cos\omega_c t| \le |x(t)|$$

Thus, if the input $x(t)$ is bounded, then the output $y(t)$ is also bounded and the system is BIBO stable.

The discrete-time system shown in Fig. 1-36 is known as the *unit delay* element. Determine whether the system is (*a*) memoryless, (*b*) causal, (*c*) linear, (*d*) time-invariant, or (*e*) stable.

(*a*) The system input-output relation is given by

$$y[n] = \mathbf{T}\{x[n]\} = x[n-1]$$

Since the output value at n depends on the input values at $n-1$, the system is not memoryless.

(*b*) Since the output does not depend on the future input values, the system is causal.

(*c*) Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$. Then

$$y[n] = \mathbf{T}\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \alpha_1 x_1[n-1] + \alpha_2 x_2[n-1]$$
$$= \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

Thus, the superposition property (*1.68*) is satisfied and the system is linear.

(*d*) Let $y_1[n]$ be the response to $x_1[n] = x[n-n_0]$. Then

$$y_1[n] = \mathbf{T}\{x_1[n]\} = x_1[n-1] = x[n-1-n_0]$$

and $$y[n-n_0] = x[n-n_0-1] = x[n-1-n_0] = y_1[n]$$

Hence, the system is time-invariant.

Fig. 1-36 Unit delay element

(*e*) Since

$$|y[n]| = |x[n-1]| \le k \qquad \text{if } |x[n]| \le k \text{ for all } n$$

the system is BIBO stable.

A system has the input-output relation given by

$$y[n] = \mathbf{T}\{x[n]\} = nx[n]$$

Determine whether the system is (*a*) memoryless, (*b*) causal, (*c*) linear, (*d*) time-invariant, or (*e*) stable.

(*a*) Since the output value at n depends on only the input value at n, the system is memoryless.

(*b*) Since the output does not depend on the future input values, the system is causal.

(*c*) Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$. Then

$$y[n] = \mathbf{T}\{x[n]\} = n\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\}$$
$$= \alpha_1 n x_1[n] + \alpha_2 n x_2[n] = \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

Thus, the superposition property is satisfied and the system is linear.

Fig. 1-38

(*d*) Let $y_1[n]$ be the response to $x_1[n] = x[n - n_0]$. Then

$$y_1[n] = \mathbf{T}\{x[n - n_0]\} = nx[n - n_0]$$

But
$$y[n - n_0] = (n - n_0)x[n - n_0] \neq y_1[n]$$

Hence, the system is not time-invariant.

(*e*) Let $x[n] = u[n]$. Then $y[n] = nu[n]$. Thus, the bounded unit step sequence produces an output sequence that grows without bound (Fig. 1-38) and the system is not BIBO stable.